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and if the x -axis coincides with diagonals 2. 4 of the quadrilateral, and 3. 6 of the octahedron, the y -axis with diagonals 1. 3 and 5. 2, and the z -axis with diagonal 1. 4, then the elements in the positive diagonal of the determinants are the respective diagonals, while all side elements vanish, and we have

$$L = \frac{1}{1!} (1. 2), \quad (1'')$$

$$A = \frac{1}{2!} (2. 4) (1. 3) \sin y_x, \quad (2'')$$

$$V = \frac{1}{3!} (3. 6) (2. 5) (1. 4) \sin y_x \sin z_{xy}. \quad (3'')$$

SOLUTIONS OF EXERCISES.

ACKNOWLEDGMENTS.

James H. Boyd 203, 208; H. W. Draughton 167, 203; E. Frisby 167; R. H. Graves 166, 167, 181, 205; A. Hall 207; J. E. Hendricks 208; J. N. James 207; Artemas Martin 208; Chas. Puryear 203; Wm. M. Thornton 167; W. O. Whitscarver 203, 208.

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SHOW that the locus of the centres of equilateral hyperbolas circumscribed to a given triangle is the nine-points circle of the triangle.

SOLUTION.

The given triangle is the triangle of reference. The circumconic

$$u\beta\gamma + v\gamma a + w\alpha\beta = 0$$

will be an equilateral hyperbola if

$$u \cos A + v \cos B + w \cos C = 0;$$

$$v\gamma + w\beta : wa + u\gamma : u\beta + va = a : b : c$$

are the equations to the centre; and the resultant of this system is

$$a\beta\gamma + b\gamma a + c\alpha\beta = aa^2 \cos A + b\beta^2 \cos B + c\gamma^2 \cos C,$$

which represents the nine-points circle. In determinant notation this resultant can be written

$$\begin{vmatrix} 0 & a & \beta & c \\ a & 0 & \gamma & b \\ \beta & \gamma & 0 & a \\ \cos C & \cos B & \cos A & 0 \end{vmatrix} = 0,$$

a form which is probably new.

[R. H. Graves.]

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P and Q are middle points of opposite edges of a tetraedron. A plane through PQ intersects two other opposite edges in M and N . Show that MN is bisected by PQ . [Asaph Hall.]

SOLUTION I.

Draw a plane (A) through an edge of the tetraedron and the opposite middle point. This plane divides the solid into equal parts, since the parts stand on the same base and have equal altitudes. Draw another plane (B) through the points $PMQN$. This plane cuts off from the equal parts two small tetraedrons which are also equal. To show this draw parallel planes through opposite edges of the given tetraedron, which will form a circumscribed parallelopipedon, the opposite edges of the tetraedron being the cross diagonals of the faces of the parallelopipedon. From this construction it follows that the points M and N are equally distant from the plane (A). Hence the small tetraedrons are equal, since they stand on equal bases and have equal altitudes. From this equality it follows that the line MN is bisected by PQ , round which the plane (B) revolves. [A. Hall.]

SOLUTION II.

Let $\epsilon_1, \epsilon_2, \epsilon_3$, and ϵ_4 be the vertices of the tetraedron; whence

$$P = \frac{1}{2}(\epsilon_1 + \epsilon_2),$$

$$Q = \frac{1}{2}(\epsilon_3 + \epsilon_4),$$

$$M = \beta\epsilon_2 + (1 - \beta)\epsilon_3,$$

and

$$N = \alpha\epsilon_1 + (1 - \alpha)\epsilon_4.$$

M, N, P, Q are coplanar; hence

$$\frac{1}{2}l(\epsilon_1 + \epsilon_2) + \frac{1}{2}m(\epsilon_3 + \epsilon_4) + n[\alpha\epsilon_1 + (1 - \alpha)\epsilon_4] = \beta\epsilon_2 + (1 - \beta)\epsilon_3.$$

Reducing, we have

$$(\frac{1}{2}l + n\alpha)\epsilon_1 + (\frac{1}{2}l - \beta)\epsilon_2 + (\frac{1}{2}m - 1 + \beta)\epsilon_3 = [n(\alpha - 1) - \frac{1}{2}m]\epsilon_4,$$

a numerical relation between four points not coplanar; hence the coefficients of the points are each equal to zero; whence $\alpha = \beta$, and

$$\frac{1}{2}(M + N) \equiv \frac{1}{2}(\alpha\epsilon_1 + \beta\epsilon_2) + \frac{1}{2}[(1 - \beta)\epsilon_3 + (1 - \alpha)\epsilon_4]$$

lies on the line PQ .

[J. N. James.]

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Prof. Graves finds the result anticipated in Byerly's Integral Calculus:

$$x = y = \frac{256a}{315\pi}.$$